THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

# MATH 2050A Tutorial 1

1. Let $S = (a, b]$ , where $a < b$. Determine the supremum and infimum of $S$.

2. Let $S = \{n/2^n : n \in \mathbb{N}\}$. Show that $supS = 1/2$. Think about what $infS$. (Hints: binomial theorem)

3. Let $A$ and $B$ be bounded non-empty subset of $\mathbb{R}$, and let $A+B := \{a+b : a \in A, b \in B\}$. Prove that $sup(A + B) = supA + supB$, and $inf(A + B) = infA + infB$.

4. Let $X$ and $Y$ be non-empty sets and let $h : X \times Y \to \mathbb{R}$ have bounded range in $\mathbb{R}$. Let $f : X \to \mathbb{R}$ and $g : Y \to \mathbb{R}$ be defined by

$$f(x) = sup\{h(x, y) : y \in Y\}, g(x) = inf\{h(x, y) : x \in X\}$$

Prove that

$$sup\{g(y) : y \in Y\} \leq inf\{f(x) : x \in X\}$$